Mark Scheme (Results)

January 2021

Pearson Edexcel International Advanced Level In Decision Mathematics (WDM11/01)

Question Number	Scheme	Marks
1.	$\left\lceil \frac{1+10}{2} \right\rceil = 6$ Diameter – reject 1 – 6	M1
	$\left\lceil \frac{7+10}{2} \right\rceil = 9$ Segment – reject 9 – 10	A1
	$\left\lceil \frac{7+8}{2} \right\rceil = 8$ Sector – reject 8	A1
	$\lceil 7 \rceil = 7$ Radius – reject Parallelogram is not in list	A1 **(4)**
		4 marks

Notes for Question 1

1M1: Choosing middle right pivot (choosing middle left 'Circumference' is M0) + an attempt at discarding/retaining half the list (condone if retaining the wrong half of the list or if only rejecting 1 – 5)

1A1: First pass correct i.e. 6th item **and** using 7 – 10 in the second pass (must not be using the 6th item in the second pass) – need not choose the 9th item or reject 9 – 10 for this mark

2A1: Second and third passes correct i.e. 9th (Segment) and 8th (Sector) items (no sticky pivots) – need not be rejecting the 8th item for this mark

3A1: CAO search complete (so rejecting 8th and 7th items) + 'not found' – must consider Radius by name or the '7th item' after rejecting sector (or stating that the seventh item is not Parallelogram). Condone if Radius is rejected on the same line that Sector is rejected (but must be after Sector). Condone those candidates that correctly state that Radius is not Parallelogram or who do not explicitly reject the Radius
e.g. '[7] = 7 Radius therefore Parallelogram is not in the list.'

Allow use of abbreviations provided clear and unambiguous. Also accept the new list of words being re-written after each pass (with or without the corresponding calculations).

For reference:

Arc Centre Chord Circle Circumference Diameter Radius Sector Segment Tangent

Question Number	Scheme	Marks
2.	Minimise $(C=)2x+3y$	B1
	$x+y\geq 85$	B1
	$y\geq 2x$	M1
	$y\leq \frac{4}{5}(x+y)$	M1
	$y\geq 2x$ and $y\leq 4x$	A1 **(5)** **5 marks**
	Notes for Question 2	

1B1: Expression correct ($2x + 3y$) together with 'minimise' or 'min' (but not 'minimum') – if 'simplified' e.g. $x + 1.5y$ then must see $2x + 3y$ at some point

2B1: CAO – any equivalent form provided integer coefficients and only one term in x and one term in y e.g. $x\geq 85-y$

1M1: y □ $2x$ where □ is any inequality or equals. Accept $2y\geq x$ for this mark

2M1: y □ $\frac{4}{5}(x+y)$ where □ is any inequality or equals – if no bracket then correct rhs must be implied by later working. y □ $4x$ where □ is any inequality or equals implies this mark. Use of % symbol only is M0 unless correctly replaced by a fraction or decimal later

1A1: Both $y\geq 2x$ **and** $y\leq 4x$ CAO – must be a single terms in x and y but allow any equivalent form provided integer coefficients e.g. $2x-y\leq 0,\quad 2y-8x\leq 0$ etc.

<table>
<tr><th>Question Number</th><th>Scheme</th><th>Marks</th></tr>
<tr><td>3. (a)</td><td>Bin 1: 2.6 0.8 1.2 0.3
Bin 2: 2.1 <u>0.9</u> <u>1.7</u>
Bin 3: <u>2.3</u> 1.8
Bin 4: 2.7</td><td>M1 <u>A1</u> A1
(3)</td></tr>
<tr><td>(b)(i)</td><td>First pass: 2.6 2.1 1.2 0.9 1.7 2.3 0.8 1.8 2.7 0.3
Second pass: 2.6 2.1 1.2 1.7 2.3 0.9 1.8 2.7 0.8 0.3</td><td>B1
B1</td></tr>
<tr><td>(b)(ii)</td><td><table><tr><td></td><td>Comparisons</td><td>Swaps</td></tr><tr><td>First pass</td><td>9</td><td>7</td></tr><tr><td>Second pass</td><td>8</td><td>4</td></tr></table></td><td>B1
B1 (4)</td></tr>
<tr><td>(c)</td><td>e.g. middle right
2.6 2.1 1.7 2.3 1.2 <u>1.8</u> 2.7 0.9 0.8 0.3 — Pivot(s) 1.8
2.6 2.1 <u>2.3</u> 2.7 1.8 1.7 1.2 <u>0.9</u> 0.8 0.3 — 2.3, 0.9
2.6 <u>2.7</u> 2.3 2.1 1.8 1.7 <u>1.2</u> 0.9 0.8 <u>0.3</u> — 2.7, (2.1), 1.2, 0.3
2.7 2.6 2.3 2.1 1.8 1.7 1.2 0.9 0.8 0.3 — Sort complete</td><td>M1
A1
A1 (3)</td></tr>
<tr><td>(d)</td><td>Bin 1: 2.7 2.3
Bin 2: 2.6 2.1 0.3
Bin 3: <u>1.8</u> <u>1.7</u> <u>1.2</u>
Bin 4: 0.9 0.8</td><td>M1 <u>A1</u> A1
(3)</td></tr>
<tr><td></td><td></td><td>13 marks</td></tr>
</table>

Notes for Question 3

a1M1: First four items placed correctly and at least seven values placed in bins. Condone cumulative totals for M1 only (the values in bold)
a1A1: First seven items placed correctly (the underlined and bold values) – any repeated/additional values then A0
a2A1: CSO (correct solution only – so no additional/repeated values)

bi1B1: CAO (first pass) – some candidates may show each comparison/swap within the first pass so take the first pass to be the list when the 0.3 is in the correct position
bi2B1: CAO (second pass) – some candidates may show each comparison/swap within the second pass so take the second pass to be the list when the 0.8 and the 0.3 are in the correct positions

ISW if completing more than two passes

bii1B1: Two correct values in the given table
bii2B1: Fully correct table completed

Mark table on page 2 of the AB (and ignore any other answers given elsewhere) but if table blank then check answer space carefully and mark this attempt instead

c1M1: Quick sort – pivot, p, chosen (must be choosing middle left or middle right – choosing first/last item as a pivot is M0). After the first pass the list must read (values greater than the pivot), pivot, (values less that the pivot). **If only choosing one pivot per iteration then M1 only. If sorting into ascending order then M0**
c1A1: First two passes correct (second pass pivot consistent with choice of pivot in first pass) – but need not be choosing pivots for the third pass
c2A1: CSO (correct solution only – all previous marks in this part **must** have been awarded) including a 'sort complete' - this could be shown by the final list being re-written or 'sorted' statement (e.g. 'done', 'complete', etc.) or each item being used as a pivot (which would therefore mean that the final list would have been written twice)

middle left:

2.6	2.1	1.7	2.3	1.2	1.8	2.7	0.9	0.8	0.3	1.2
2.6	2.1	1.7	2.3	1.8	2.7	**1.2**	0.9	0.8	0.3	1.7, 0.8
2.6	2.1	2.3	1.8	2.7	**1.7**	**1.2**	0.9	**0.8**	0.3	2.3, (0.9), (0.3)
2.6	2.7	**2.3**	2.1	1.8	**1.7**	**1.2**	0.9	**0.8**	0.3	2.6, 2.1
2.7	**2.6**	**2.3**	**2.1**	1.8	**1.7**	**1.2**	0.9	**0.8**	0.3	Sort Complete

Two Special Cases for (c): Case I: Those that perform a quick sort on the original list can score M1 only. Case II: Those that perform a quick sort on 2.6 2.1 1.7 2.3 1.2 1.8 2.7 (so not including the last three numbers in the list) can score M1A1 only

No misreads in (d) – mark according to scheme in all cases

d1M1: First four items placed correctly and at least seven values placed in bins – condone cumulative totals for M1 only (the bold values)
d1A1: First seven items placed correctly (the underlined and bold values) – any repeated/additional values is A0
d2A1: CSO (so no additional/repeated values)

Question Number	Scheme	Marks
4. (a)	e.g. in the practical problem each vertex must be visited at least once. In the classical problem each vertex must be visited exactly once	B2, 1, 0 **(2)**
(b)	NNA starting at A: A – B – D – F – C – G – E – A 25 + 24 + 35 + 27 + 29 + 31 + 35 = 206 (km)	B1 B1 **(2)**
(c)	The better upper bound is the one starting at D as it is smaller	B1dep **(1)**
(d)(i)	Prim (starting at A): AB, BD, BE, EF, CF RMST weight = 25 + 24 + 27 + 28 + 27 = 131	M1 A1
(d)(ii)	131 + 29 (CG) + 31 (EG) = 191 (km)	M1 A1 **(4)**
(e)	The better lower bound is the one found by deleting G as this is the larger of the two	B1dep **(1)**
(f)	191 ≤ optimal distance ≤ 203	B1ft B1 dep **(2)**
		12 marks

Notes for Question 4

a1B1: Understands the difference is connected to the number of times each vertex may be visited – condone 'point' (oe) for vertex (must refer to both problems in their answer but not necessarily by name)
a2B1: Correctly identifies which is classical (each node visited 'exactly once' or 'once') and which is practical (each node visited 'at least once' but B0 for 'more than once' oe – it must be clear that for the practical case that a node may be visited more than once but not necessarily more than once). Must use correct language (e.g. vertex or node) but condone singular/plural confusion e.g. vertex for vertices, or poor spelling (in this part a mark of B0B1 is not possible)

b1B1: Correct nearest neighbour route starting at A (must return to A) – possibly stated in terms of arcs e.g. AB, BD, DF, CF, CG, EG, EA
b2B1: CAO (206) on length of route

c1B1dep: CAO dependent on the correct UB in (b) – allow 'yes it is' (as question asks, 'state whether this (an upper bound of 203) is a better upper bound than the answer to (b)') **and** with some indication that this value is smaller than the one in (b) e.g. '203 < 206 so yes it is' scores B1

di1M1: Must be using Prim's algorithm not NNA. First three arcs (or all 6 nodes / or numbers across the top of the matrix) selected correctly. First three arcs are AB, BD, BE, first six nodes are A, B, D, E, F, C and so numbers across the matrix would be 1, 2, 6, 3, 4, 5. Award M1 only for a correct tree with either no working or if starting at a different node than A
di1A1: CAO (order of arc selection clear) – in terms of arcs only for this mark - AB, BD, BE, EF, CF – condone those that state AB, BD, BE, EF, CF, CG, EG **or** AB, BD, BE, EF, CF, EG, CG (these candidates are most likely adding on the two smallest arcs incident to G for the next part of the question)

dii2M1: Adding two least weighted arcs (CG(29) + EG(31)) to the length of their answer from d(i) (where $100 \leq d(i) \leq 160$) - condone if parts (d)(i) and (d)(ii) are combined together as a single part (d)
dii2A1: CAO (191)

e1B1dep: CAO dependent on the correct LB in (d)(ii) – allow 'no it isn't' (as question asks, 'state whether this (a lower bound of 188) is a better lower bound than the answer to (d)(ii)') and with some indication that this value is smaller than the one in (d)(ii) e.g. '188 < 191 so no it isn't' scores B1

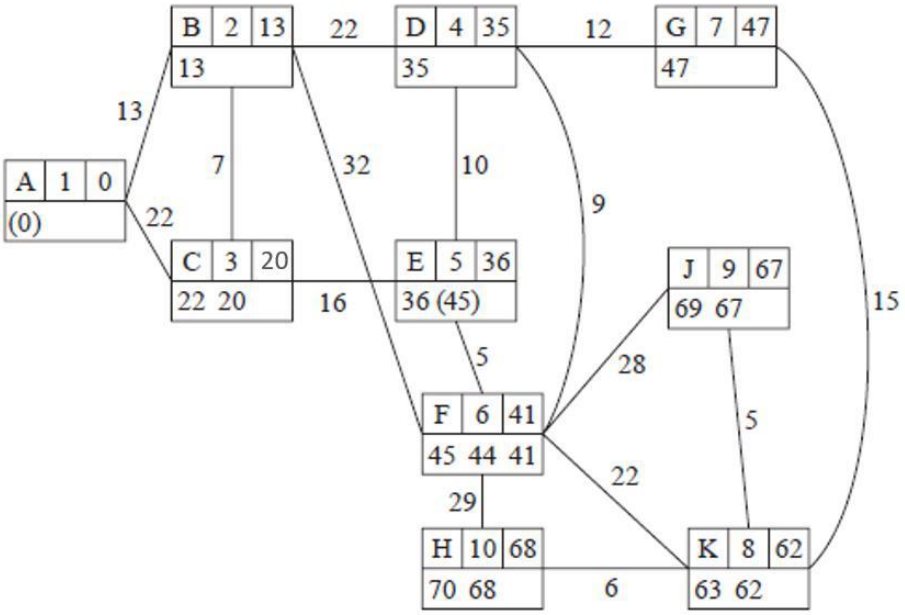
If the candidate's answer to (b) is less than 188 then no marks can be awarded in (f)

f1B1ft: Their numbers correctly used, accept any inequalities or any indication of an interval from

their largest of the two values (188 or d(ii)) to their smallest of the two values (203 or (b))

e.g. condone for B1 only 203 – 191 = 12

f2B1dep: This mark is dependent on the previous B mark - CAO including correct inequalities (accept either $191 \leq$ optimal distance ≤ 203 or $191 <$ optimal distance ≤ 203) or equivalent notation e.g. [191, 203] or (191, 203]

Question Number	Scheme	Marks
5. (a)	 Shortest path: A B D G K H Length: 68 (miles)	M1 A1 (ABCDE) A1 (FGK) A1ft (JH) A1 A1ft **(6)**
(b)	Route from F to K via A: F E C B A B D G K Length: 41 + 62 = 103 (miles)	B1 B1ft **(2)**
(c)	AJ + CE = 67 + 16 = 83 AC + EJ = 20 + 32 = 52 AE + CJ = 36 + 48 = 84 Repeated arcs: AB, BC, EF, FK, JK Length: 253 + 52 = 305 (miles)	M1 A1 A1 A1 A1 A1ft **(6)**
(d)	Vertex F: 4 times	B1 **(1)**
(e)	(Start at D and) finish at C	B1 **(1)**
(f)	Difference = 305 – (253 + 10) = 42 (miles)	B1 **(1)**
		17 marks

Notes for Question 5

In (a) it is important that all values at each node are checked very carefully – the order of the working values must be correct for the corresponding A mark to be awarded e.g. at F the working values must be 45 44 41 in that order (so 45 41 44 is incorrect)
It is also important that the order of labelling is checked carefully. The order of labelling must be a strictly increasing sequence – so 1, 2, 3, 3, 4, … will be penalised once (see notes below) but 1, 2, 3, 5, 6, … is fine. Errors in the final values and working values are penalised before errors in the order of labelling

a1M1: A larger value replaced by a smaller value in at least two of the working value boxes at either C, F, K, J, or H
a1A1: All values at A, B, C, D and E correct and the working values in the correct order (including order of labelling) – if a working value of 45 appears at E then it must appear after the 36 so therefore 45 36 at E (in this order) is A0
a2A1: All values at F, G and K correct and the working values in the correct order (F, G and K must be labelled in that order and F must be labelled after A, B, C, D and E)
a3A1ft: All values in J and H correct on the follow through and the working values in the correct order. Penalise order of labelling only once per question. To follow through J check that the working value at J follows from the candidate's final values from their feeds into J (which will mostly likely come from nodes F and K (in the order in which the candidate has labelled them)) and that the final value, and order of labelling, follows through correctly. Repeat this process for H (which will possibly have working values from F and K with the order of these values determined by the candidate's order of labelling at F and K)
a4A1: CAO (ABDGKH or AB, BD, DG, GK, KH)
a5A1ft: Follow through on their final value at H **only** (condone lack of units) so if answer given as 68 but final value at H is not 68 then A0

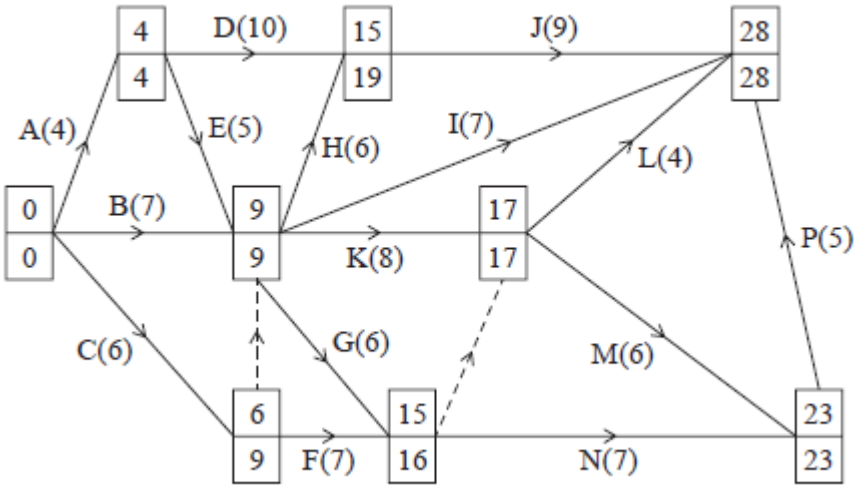
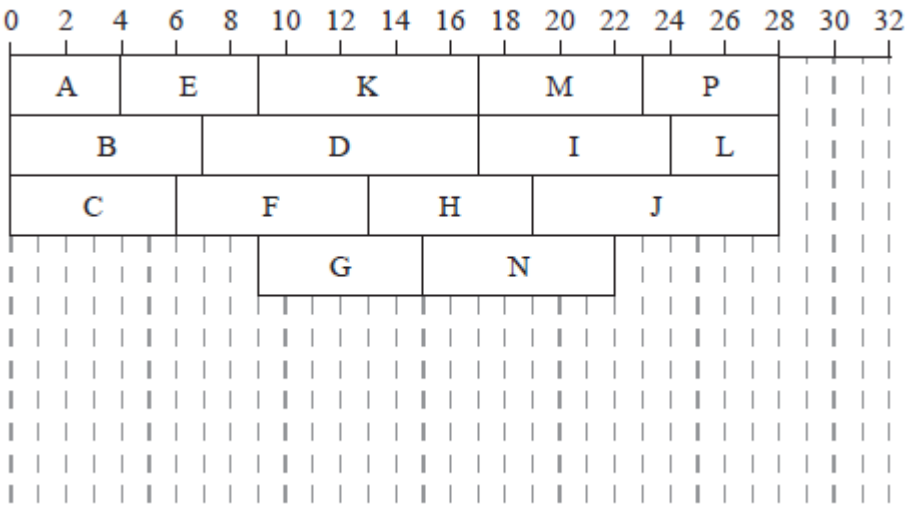
b1B1: CAO (FECBABDGK or FE, EC, CB, BA, AB, BD, DG, GK)
b2B1ft: Follow through their final value at F + their final value at K **or** 103

c1M1: Correct three distinct pairings of the correct four odd nodes of A, C, E and J
c1A1: **Any** row correct including pairing **and** total
c2A1: **Any** two rows correct including pairings **and** totals
c3A1: **All** three rows correct including pairings **and** totals
c4A1: CAO correct edges clearly stated and not just in their working as AB, BC, EF, FK and JK – must be these arcs
c5A1ft: Follow through their value of their smallest pairing total + 253

d1B1: CAO (4 only)

e1B1: CAO (C)

f1B1: CAO (42)

Question Number	Scheme	Marks
6. (a) and (b)		B1 B1 B1 **(3)** M1 A1 M1 A1 **(4)**
(c)	Critical activities: A, E, K, M, P	B1 **(1)**
(d)	Lower bound = $\frac{97}{28}$ = 3.46… so 4 workers	M1 A1 **(2)**
(e)	e.g. 	M1 A1 A1 **(3)**
		13 marks

Notes for Question 6

a1B1: Any two of the five arcs (G, H, I or the two dummies) drawn correctly (from correct vertex to correct vertex) – activities labelled with the correct letter (but condone no or wrong arrows) and the dummies must be shown as dashed lines (or labelled as 'dummy') with no weight (but condone no or wrong arrows)
a2B1: Four of the five arcs (G, H, I and the two dummies) drawn correctly – activities must be labelled with the correct letter (but condone no or wrong arrow) and the dummies must be shown as dashed lines (or labelled 'dummy') with no weight (but condone no or wrong arrows)
a3B1: CSO - all three activities (G, H and I) and the two dummies drawn correctly with no extras. Activities must be labelled with the correct letter and weights. The activities **and** dummies (as dashed lines with zero weight) must have the correct arrows (**do check carefully that all arrows are present**)
In (a) condone for full marks activities which are shown as dashed lines provided they are labelled with the correct letter. Condone lack of (or incorrect) weights on the activity arcs for the first two marks only.

b1M1: All top boxes complete (condone lack of 0 for the M mark only), values generally increasing in the direction of the arrows ('left to right'), condone one 'rogue' value (if values do not increase in the direction of the arrows then if one value is ignored and the remaining values do increase in the direction of the arrows then this is considered to be a single rogue value). Note that all values in the top boxes could be incorrect but it can still score the M mark if the values are **increasing** in the way stated above – **this mark is dependent on the first mark having being awarded in (a)**
b1A1: CAO – all values correct in the top boxes
b2M1: All bottom boxes complete (condone lack of 28 and/or 0 for the M mark only), values generally decreasing in the opposite direction of the arrows ('right to left'), condone one 'rogue' – **this mark is dependent on the first mark having being awarded in (a)**
b2A1: CAO – all values correct in the bottom boxes
For full marks in (b) all three activities (G, H and I) and the two dummies must have been added correctly in (a) – condone lack of arrows only. If all values in the bottom and top boxes are correct but any arc or dummy is missing or incorrect then award M1A1M1A0 – if all values are not correct (and some arcs are missing) then mark to the scheme above

c1B1: CAO (A, E, K, M, P only)

d1M1: Attempt to find lower bound: (a value in the interval [87 – 107] / their finish time) **or** showing the summing of **all** 15 activities divided by their finish time **or** (as a minimum) an awrt 3.5
d1A1: CSO – either a **correct** calculation seen **or** awrt 3.5 **then** 4 (with no incorrect working seen). An answer of 4 with no working scores M0A0
e1M1: Not a cascade (Gantt) chart. 5 'workers' used at most and at least 11 activities placed
e1A1: 4 workers. All 15 activities present (just once). Condone at most two errors. An activity can give rise to at most three errors; one on duration, one on time interval and only one on IPA
e2A1: 4 workers. All 15 activities present (just once). No errors

Activity	Duration	Time	IPA
A	4	0 – 4	-
B	7	0 – 9	-
C	6	0 – 9	-
D	10	4 – 19	A
E	5	4 – 9	A
F	7	6 – 16	C
G	6	9 – 16	B, C, E
H	6	9 – 19	B, C, E

Activity	Duration	Time	IPA
I	7	9 – 28	B, C, E
J	9	15 – 28	D, H
K	8	9 – 17	B, C, E
L	4	17 – 28	F, G, K
M	6	17 – 23	F, G, K
N	7	15 – 23	F, G
P	5	23 – 28	M, N

Question Number	Scheme	Marks
7. (a)	$x+y\leq 8$ (oe) $5y\geq x+k$ (oe) $y=-\frac{8}{4}x+8$ (or any inequality symbol replacing 'equals') $2x+y\geq 8$ (oe)	B1 B1 M1 A1 **(4)**
(b)	$P=5x+ky$ If $(0,8)$ is the optimal vertex then $k=\frac{19}{4}$ (= 4.75) Other possible optimal vertex is the point of intersection of $x+y=8$ and $5y=x+k$ and attempting to solve simultaneously **or** attempt to express x and k in terms of y **or** attempt to express y and k in terms of x $\left(\frac{40-k}{6},\frac{8+k}{6}\right)$ **or** stating $\begin{matrix}y=8-x\\k=40-6x\end{matrix}$ **or** stating $\begin{matrix}x=8-y\\k=6y-8\end{matrix}$ $5\left(\frac{40-k}{6}\right)+k\left(\frac{8+k}{6}\right)=38$ **or** $5x+(40-6x)(8-x)=38$ **or** $5(8-y)+(6y-8)y=38$ $k^2+3k-28=0\Rightarrow(k-4)(k+7)=0$ **or** $6x^2-83x+282=0\Rightarrow(6x-47)(x-6)=0$ then $k=\ldots$ **or** $6y^2-13y+2=0\Rightarrow(6y-1)(y-2)=0$ then $k=\ldots$ $k=4$ If $k=4$ then $(6,2)\rightarrow P=38$ and $(0,8)\rightarrow P=32$ If $k=\frac{19}{4}$ then $(0,8)\rightarrow P=38$ and $\left(\frac{47}{8},\frac{17}{8}\right)\rightarrow P=\frac{1263}{32}(=39.468...)$ and so $k=4$ only	B1 M1 A1 dM1 ddM1 A1 A1 **(7)**
		11 marks

Notes for Question 7

a1B1: CAO ($x + y \leq 8$) – any equivalent inequality (but not strict inequality)

a2B1: CAO ($5y \geq x + k$) – any equivalent inequality (but not strict inequality)

a1M1: **Correct** equation (or with any inequality symbol) of the line through (0, 8) and (4, 0)

a1A1: CAO (any equivalent form with three terms only – condone coefficients that are not integers) – any equivalent inequality (but not strict inequality)

b1B1: $k = \frac{19}{4}$ (oe exact value) seen

b1M1: Correct method for solving the correct pair of simultaneous equations ($x + y = 8$ and $5y = x + k$) to find both *x* and *y* in terms of *k* **or** attempt to express *x* and *k* in terms of *y* only **or** attempt to express *y* and *k* in terms of *x* only (this mark can be implied by a **correct** equation in either *k*, *x* or *y* only)

b1A1: CAO for coordinates of possible optimal vertex in terms of *k* – allow correct unsimplified, for example, $x = \frac{40-k}{6}, y = 8 - \frac{40-k}{6}$ **or** for $\begin{matrix} y = 8 - x \\ k = 40 - 6x \end{matrix}$ **or** for $\begin{matrix} x = 8 - y \\ k = 6y - 8 \end{matrix}$ (this mark can be implied by a **correct** equation in either *k*, *x* or *y* only)

b2dM1: Setting up an equation in either *k*, *x* or *y* only using the intersection point of $x + y = 8$ and $5y = x + k$, together with 5*x*+ *ky* and 38 (dependent on previous M mark)

b3ddM1: Solving a three term quadratic in *k*, *x* or *y* and finding at least one positive value of *k* (dependent on both previous M marks) – if the method for solving their quadratic is not shown then this mark can be implied if their value of *k* satisfies their equation. If solving using the quadratic formula then they must be using the correct formula with their values, if using factorisation then when expanding their brackets must give two terms of their 3-term quadratic. Note that $5\left(\frac{40-k}{6}\right) + k\left(\frac{8+k}{6}\right) = 38$ (oe) $\Rightarrow k = 4$ can imply this and the next A mark

b2A1: $k = 4$ (ignore mention of other values of *k*, e.g. $k = -7$) but this value must have come from correct working

b3A1: Clear rejection of *k* = 19/4 (by showing that for this value of *k*, *P* > 38) **and** <u>evidence</u> of the second root or factor in *k* (e.g. stating at some point that $k = -7$ or $(k+7)(k-4) = 0$ seen) which is also rejected (possibly done implicitly so e.g. $(k+7)(k-4) = 0 \Rightarrow k = 4$ would be sufficient), **and** stating *k* = 4 only (so not giving more than one value for *k*) – dependent on all previous marks in (b)

The correct value of *k* with no working scores no marks – correct value with minimal working then please send to review